Discussion [0b10] [2] [0x2]

Limits

(a) **What is the biggest number that can be represented with two decimal digits?**

99

(b) **What is the biggest number that can be represented with three binary digits?**

The greatest number that can be represented with three binary digits is 0b111. In decimal, this is $7 = 1 \times 2^0 + 1 \times 2^1 + 1 \times 2^2$

Note: Another way to do this is to notice that 0b111 is just 1 less than 0b1000, which is equal to 2^3. That means the greatest number that can be represented with three binary digits is $2^3 - 1 = 7$.

(c) **What is the biggest number that can be represented with four hexadecimal digits?**

The greatest number that can be represented with four hex digits is 0xFFFF. Following the note from part (b), we get $16^4 - 1 = 65535$.

(d) **How many different numbers can you represent using three binary digits?**

8 distinct numbers. The maximum number we can represent is 7, and the minimum number is 0.

Conversion

(a) Convert the following binary numbers into decimal.

11001 → **25**

1001001 → **73**

(b) Convert the following decimal numbers into binary.

12 → **0b1100**

64 → **0b1000000**

127 → **0b1111111**

(c) Convert the following binary numbers into hexadecimal.

10011001 → **0x99**

11110111 → **0xF7**

1100000011111111101110 → **0xCOFFEE**

(d) Fill in the blanks.

Decimal	Binary	Hexadecimal
12	**0b1100**	C
5	**0b101**	5
11	1011	**0xB**
25	11001	**0x19**
17	10001	11
27	11011	1B
8	**0b1000**	**8**
14	1110	**0xE**
30	**0b11110**	1E
73	**0b1001001**	49

Challenge Problems

(a) The original Pokemon are numbered 1-150. We want to store a binary encoding for all original Pokemon where each Pokemon has a binary code equivalent to their decimal number. How many bits do we need to use?
8 bits, since 150 can be represented with 8 bits.

(b) What is the encoding for Pikachu (#25)?
0b11001

(c) Ternary utilizes base 3 instead of base 2. For example, 10 in ternary is

equivalent to 3 in decimal. Imagine that we wanted to store a ternary encoding for all 150 Pokemon where each Pokemon has a ternary code equivalent to their decimal number. What is the ternary encoding for Pikachu (#25)?

221.

Using the method shown in lecture for converting decimal to other bases, we can create the following table:

Power	3^3	3^2	3^1	3^0
Value	27	9	3	1
Ternary digit				

Now, we notice that the greatest value listed in our table that is still less than 25 is 9. When we divide 25 by 9, we get 2 (with some remainder). Then, we will put a 2 in the ternary digit under 9, and our new value we are working with is 25 - (9 x 2) = 7. Our table then looks like this:

Power	3^3	3^2	3^1	3^0
Value	27	9	3	1
Ternary digit		2		

Now, we repeat the process with the value of 7. 3 is the greatest value that is less than 7, 7 divided by 3 gives us 2. So we put a 2 in the ternary digit under 3, and we continue with a value of 7 - (3 x 2) = 1. Our table looks like this:

Power	3^3	3^2	3^1	3^0
Value	27	9	3	1
Ternary digit		2	2	

Now, our new value is 1. The greatest value that's less than 1 is 1, and 1 divided by 1 is 1. Our new value is now 1-1 = 0, which means we are done. Our table looks as follows:

Power	3^3	3^2	3^1	3^0
Value	27	9	3	1
Ternary digit		2	2	1

And our answer is 221.

Here is how we would get the same answer using the method shown in discussion:

New Base	Value	Remainder
3	25	

Every step of the way, we divide our value by the new base, and then record the remainder. So, at the next step, our table would look like this:

New Base	Value	Remainder
3	25	
3	8	1

We continue this process until our value is equal to 0.

New Base	Value	Remainder
3	25	
3	8	1
3	2	2
3	0	2

To get our answer, we read the remainder column upwards, so our answer is 221.